MEC524 Chapter 4: System performance analysis

Cheat sheet, handbook § 4.0–§ 4.6

Why it matters

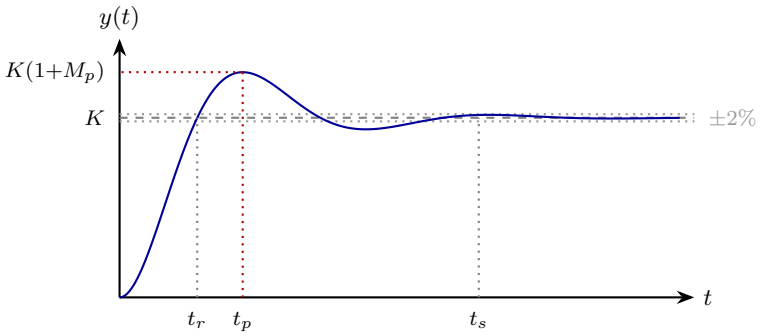
A model says how a machine moves; a specification says how it should. This chapter reads time constants, overshoot, settling time and steady error off a transfer function, decides with Routh whether a loop is stable, and shows how the poles move as the gain is turned up.

Start here

Find the poles first. A single real pole behaves like a first-order lag; a complex pair gives ζ and ω_n, and from them the specifications. Check stability before any specification means anything.

One picture

The step response of a second-order system and the numbers that describe it. The animation swings the poles round a circle of fixed ω_n as ζ rises, and the overshoot and settling time follow. Animated: `Decks/ch4-anim/MEC524-ch4-zeta.gif`.

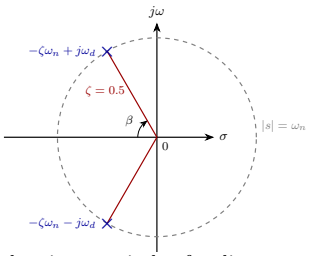


Which formula?

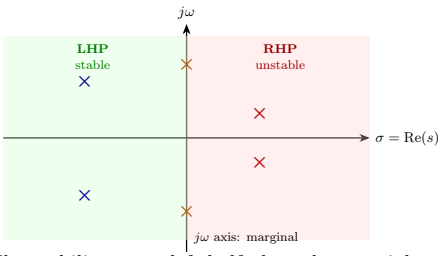
Read the situation first, then pick the formula. The numbers in brackets are the handbook's equation numbers.

You see...	Use	It holds when
One time constant	$G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{K}{\tau s + 1}$ (4.2)	τ is the time to 63.2 % of the final value.
	$y(t) = K\left(1 - e^{-t/\tau}\right), \quad t \geq 0$ (4.3)	The unit-step response.
A second-order plant	$G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{K\,\omega_n^2}{s^2 + 2\zeta\omega_n\, s + \omega_n^2}$ (4.6)	Standard form: read ζ and ω_n off the denominator.
	$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$ (4.7)	Complex for $\zeta < 1$, real and repeated at $\zeta = 1$.
How much it overshoots	$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$ §4.2	Depends on ζ alone.
How long until it settles	$t_s = \dfrac{4}{\zeta\omega_n}$ §4.2	The 2 % band; set by the real part of the poles.
The error left at the end	$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \dfrac{s\,R(s)}{1 + G(s)}$ (4.10)	Unity feedback, stable loop; the system type decides which inputs it tracks.
Is the loop stable?	$b_1 = \dfrac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}, \quad b_2 = \dfrac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}, \quad \ldots$ (4.12)	Routh array: each sign change in the first column is one right-half-plane pole.
Where the poles go as K grows	$\theta_a = \dfrac{(2k+1)\times 180^\circ}{n - m}, \quad k = 0, 1, \ldots, n - m - 1$ (4.17)	Asymptote angles, for $n - m$ branches going to infinity.
	$\sigma_a = \dfrac{\sum_j p_j - \sum_i z_i}{n - m}$ (4.18)	Where the asymptotes meet the real axis.

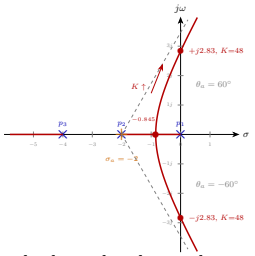
Reading the pictures



The poles sit on a circle of radius ω_n; the angle from the negative real axis sets ζ.



The stability map: left half-plane decays, right half-plane grows, the imaginary axis rings forever.



A root locus: the branches leave the open-loop poles and two of them cross into the right half-plane at the critical gain.

Ways to lose marks

Specifications need stability. A step-response specification means nothing for an unstable loop: check every pole first.
Two different knobs. Overshoot depends on ζ only; settling time on $\zeta\omega_n$.
A zero in the Routh column. Replace it by a small ε, or use the auxiliary polynomial for a whole row of zeros.
System type. Count the integrators in the open loop $G(s)$, not in the closed loop.
Asymptotes. They need $n - m$: open-loop poles minus open-loop zeros.

Where next. Handbook: § 4.0–§ 4.6 for the worked examples and tutorials. **Slides:** the Chapter 4 deck, with every animation.