MEC524 Chapter 2: System model representation

Cheat sheet, handbook § 2.0–§ 2.6

Why it matters

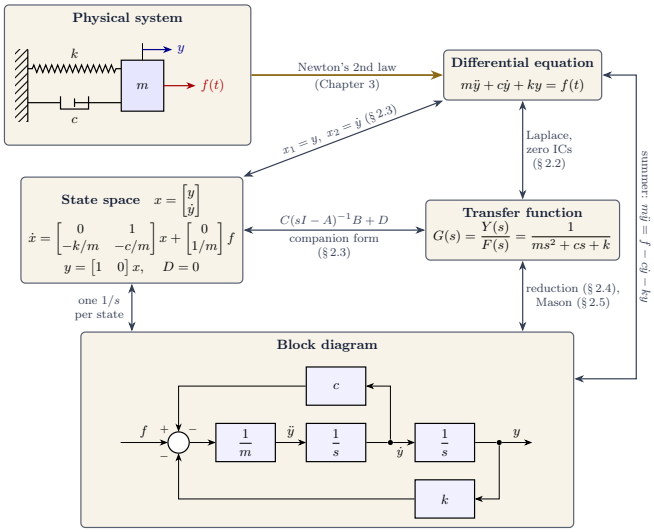
One machine can be written four ways: a differential equation, a transfer function, state space, and a block diagram or signal-flow graph. Each suits a different job, and moving between them without losing anything is the skill this chapter builds.

Start here

Linearise first if the physics is not linear. Then use the form the question asks for, and check at the end that the characteristic equation, the poles and the DC gain come out the same in every form.

One picture

Four representations of one plant, each an exact translation of the others. The animation reduces a block diagram move by move and runs the response after every move: it never changes. Animated: `Decks/ch2-anim/MEC524-ch2-reduce.gif`.

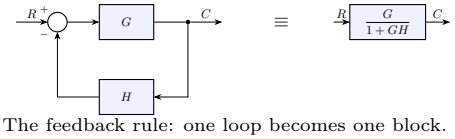


Which formula?

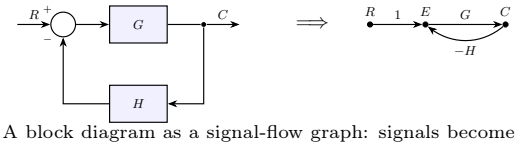
Read the situation first, then pick the formula. The numbers in brackets are the handbook's equation numbers.

You see...	Use	It holds when
A nonlinear element	$f(x,u) \approx f(x_0,u_0) + \left.\frac{\partial f}{\partial x}\right\|_0 (x - x_0) + \left.\frac{\partial f}{\partial u}\right\|_0 (u - u_0)$ (2.2)	Small deviations about the operating point only.
Input–output form	$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0}$ (2.3)	Zero initial conditions; the denominator set to zero is the characteristic equation.
Where a step response settles	$K_{\mathrm{DC}} = G(0) = \frac{b_0}{a_0}$ (2.5)	Only for a stable system: the final value of a unit-step response.
Internal variables, several inputs	$\dot{x}(t) = A\,x(t) + B\,u(t)$ (2.7)	One first-order equation per state; the number of states is the order.
	$y(t) = C\,x(t) + D\,u(t)$ (2.8)	D is zero unless the input reaches the output directly.
State space to a transfer function	$G(s) = \frac{Y(s)}{U(s)} = C\,(sI - A)^{-1}\,B + D$ (2.9)	The poles of $G(s)$ are the eigenvalues of A.
A feedback loop to reduce	$\frac{C}{R} = \frac{G}{1 \pm GH}$ (2.11)	Plus in the denominator for negative feedback.
Many loops, no reduction	$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k\,\Delta_k$ (2.12)	Δ subtracts every loop gain and adds the products of loops that do not touch.

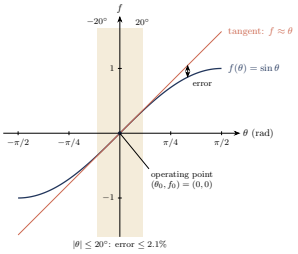
Reading the pictures



The feedback rule: one loop becomes one block.



A block diagram as a signal-flow graph: signals become nodes, blocks become branches, and the signs move onto the branches.



Linearisation: the tangent at the operating point stands in for the curve, and is only good close to it.

Ways to lose marks

Moving a summer or a pickoff. Moving it past a block needs a compensating block. Check by tracing one signal through, before and after.

Non-touching loops. Only loops that share no node multiply in Δ.

Too few states. An nth-order equation needs n states, so a second-order plant has a 2×2 A.

The feedback sign. $G/(1 + GH)$ is negative feedback; mixing up the signs flips stability.

Linear only near the point. A linearised model is good for small deviations only.

Where next. Handbook: § 2.0–§ 2.6 for the worked examples and tutorials. **Slides:** the Chapter 2 deck, with every animation.