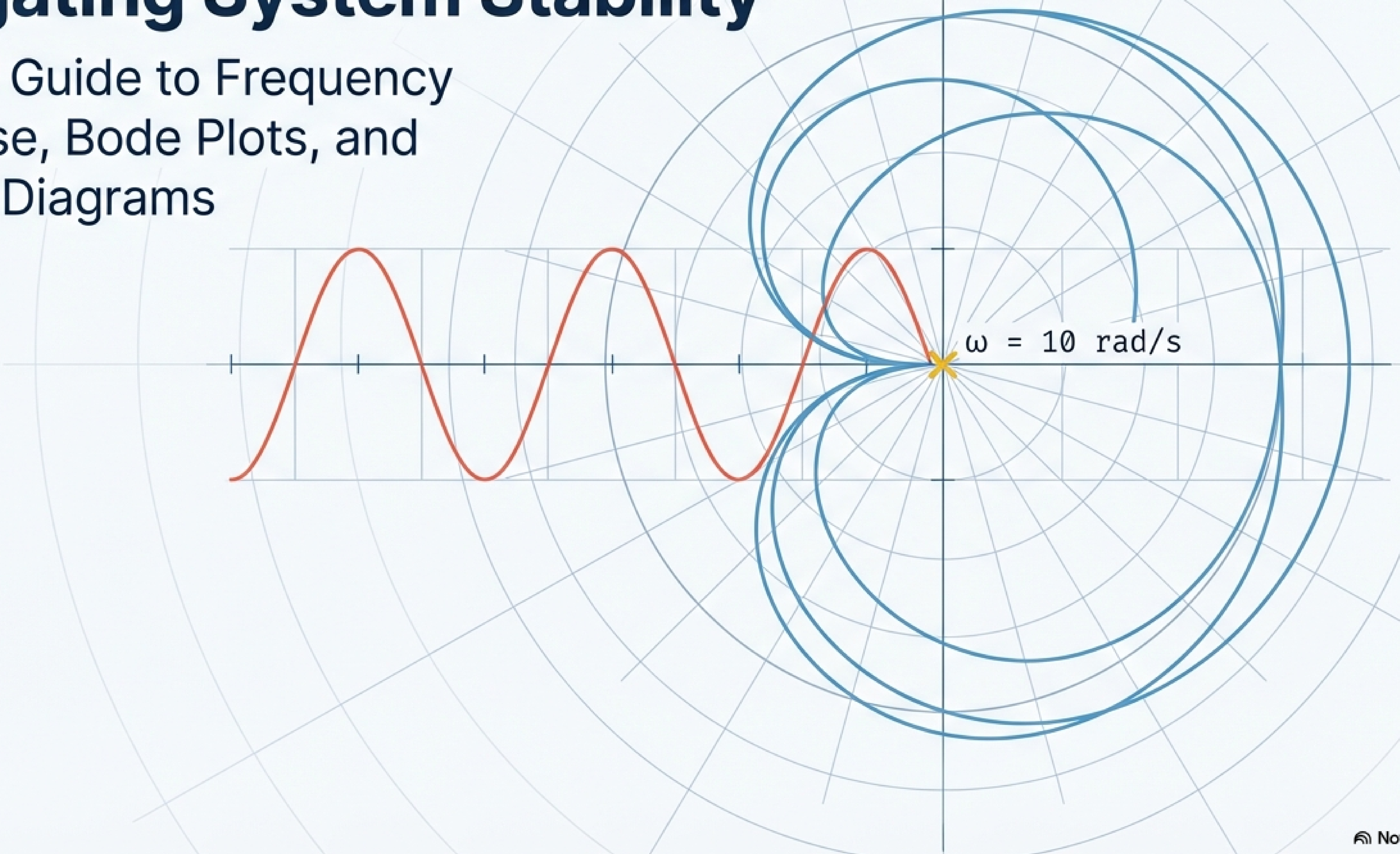
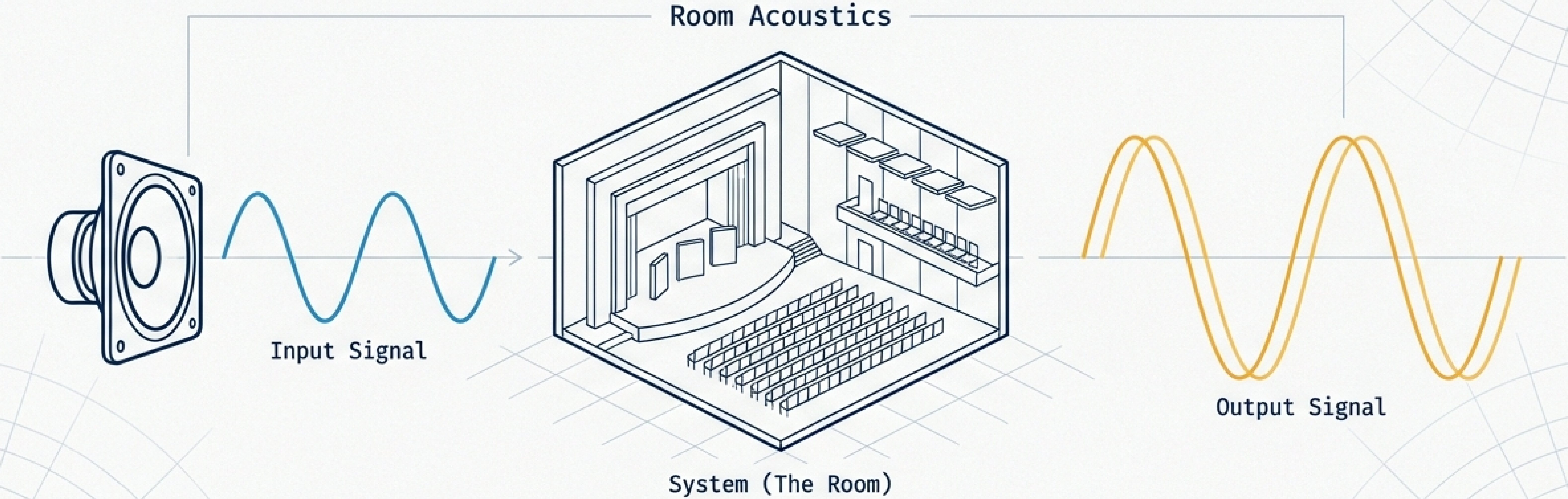
Navigating System Stability

A Visual Guide to Frequency Response, Bode Plots, and Nyquist Diagrams



The Core Diagnostic: Frequency Response

Frequency response analysis examines how a system responds to sinusoidal inputs across a range of frequencies without requiring a complete mathematical model—acting as a diagnostic tool using experimental data.



Gain $|G(j\omega)|$

How much the system amplifies or attenuates the input signal. (Measured in V/V, A/A, W/W, or dB)

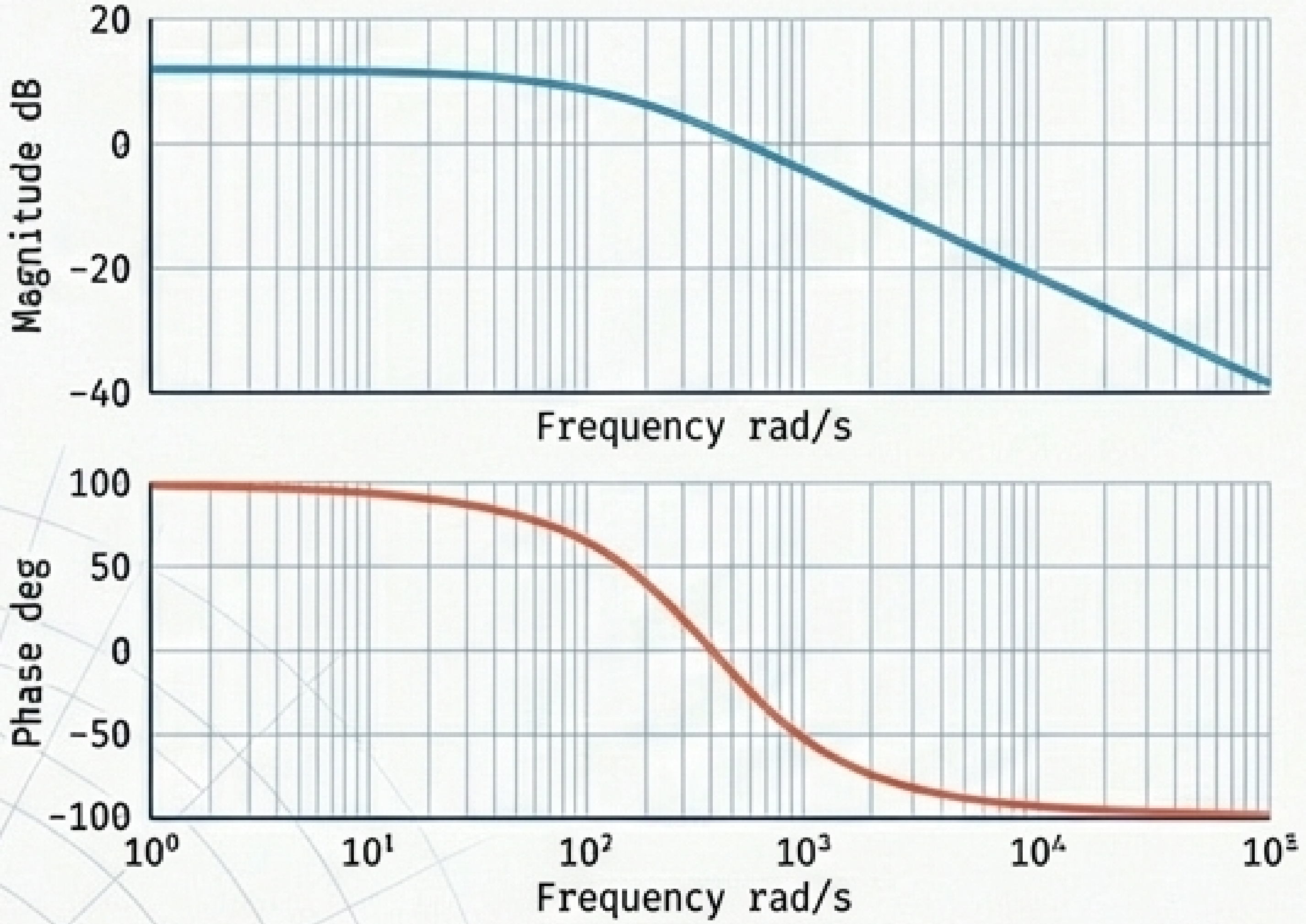
Phase $\phi(j\omega)$

How much the system delays or advances the input signal. (Measured in degrees)

Two Instruments for Navigation

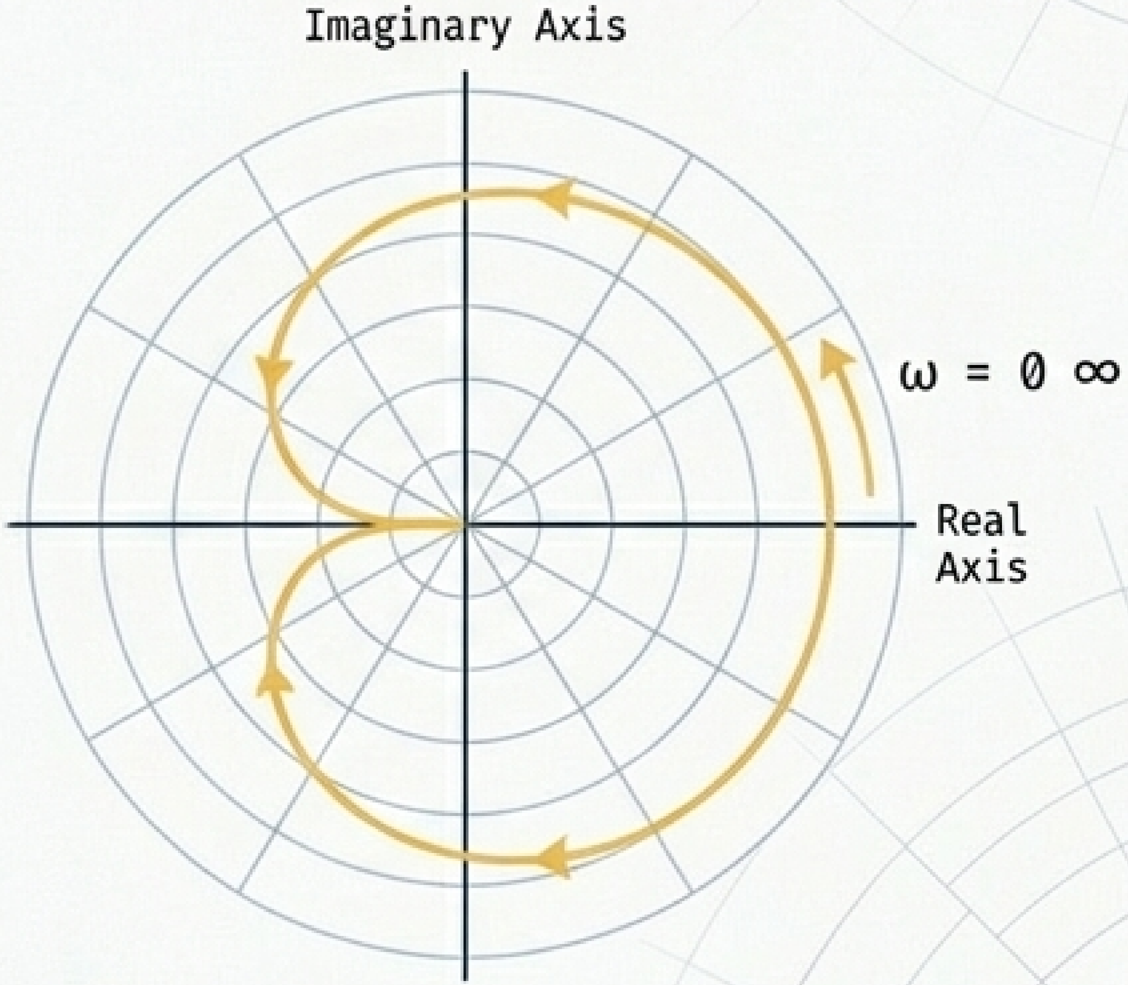
Developed in the 1930s-1940s by Harry Nyquist, Hendrik Bode, and Nathaniel Nichols, these two graphical methods serve as the primary lenses through which engineers view control system stability.

Instrument 1: The Bode Lens



Deconstructs response into separate Magnitude and Phase graphs against a logarithmic frequency scale. Best for individual component design.

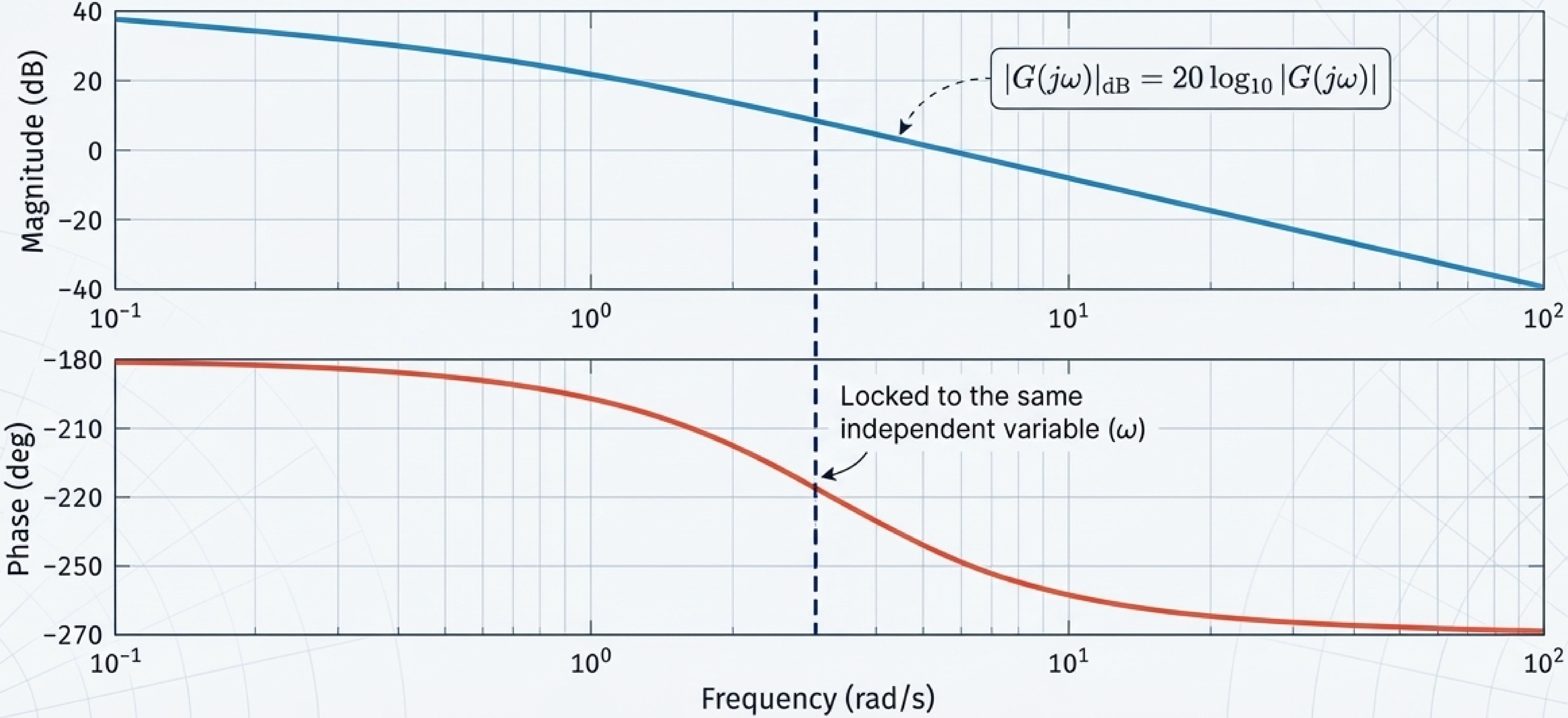
Instrument 2: The Nyquist Lens



Combines Magnitude and Phase into a single parametric curve mapped from $\omega = 0$ to ∞. Best for direct system stability assessment.

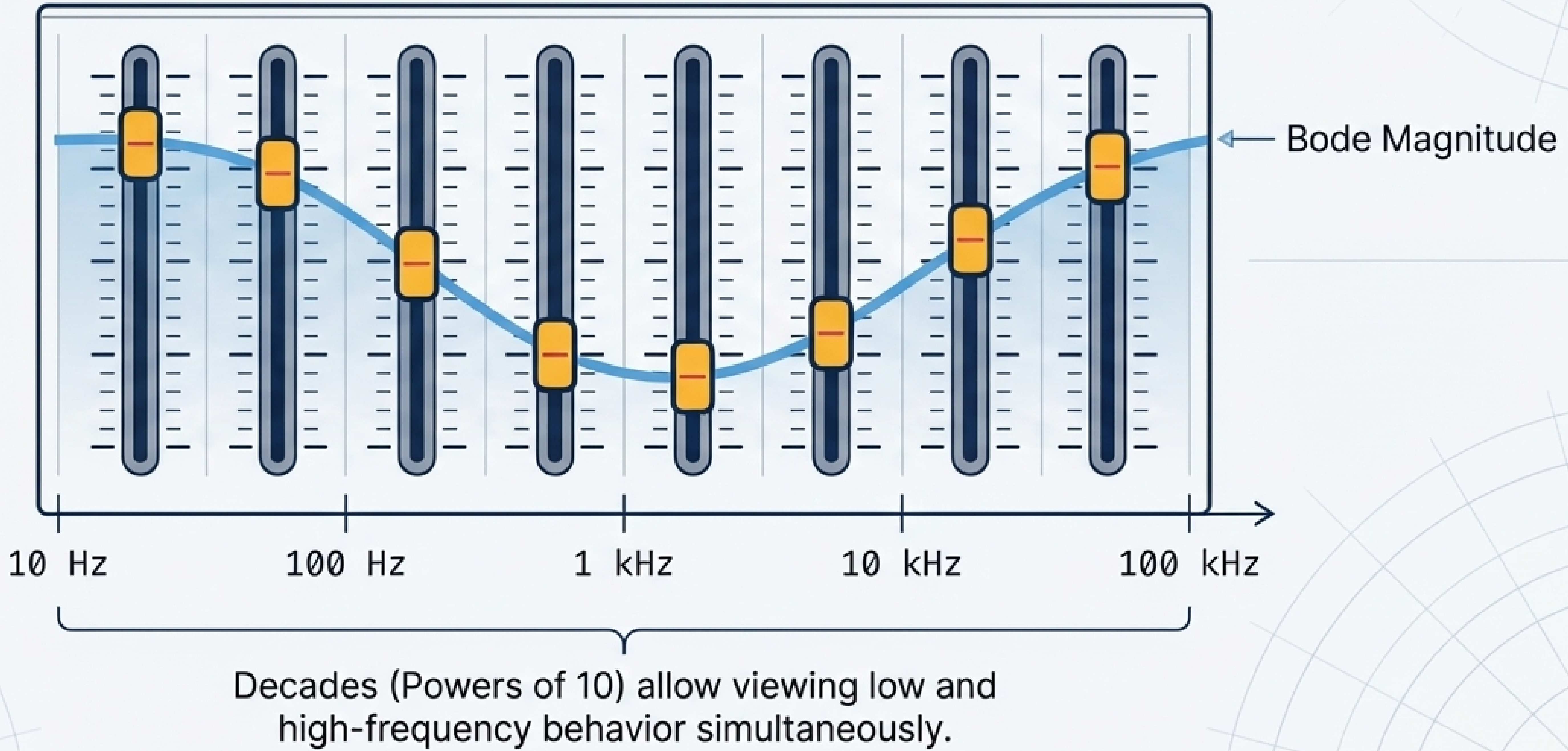
Instrument 1: The Bode Plot

The Bode plot separates frequency response into two distinct views, allowing engineers to track exactly how amplitude and timing shift as frequency increases.



The Graphic Equalizer Metaphor

A linear scale displaying a gain of 100 cannot effectively show the difference between 0.10 and 0.05. Bode plots use a logarithmic scale because physical systems scale proportionally across decades, not linear intervals.



Constructing the Bode Loci

The asymptotic Bode plot uses straight-line approximations to chart system behavior. Each pole and zero alters the trajectory of the plot at specific corner frequencies.

1. Normalize

Factor transfer function into standard form: $(s/\omega_{bf} + 1)$

2. Locate Breaks

Identify break frequencies (ω_{bf}) for each pole/zero.

3. Start Point

Determine magnitude & phase one decade below lowest ω_{bf}.

4. Tabulate Gradients

Poles:
-20 dB/dec & -45°/dec
Zeros:
+20 dB/dec & +45°/dec

5. Plot Loci

Cumulatively add gradients at each break frequency.

6. Extract Margins

Read Gain Margin (GM) & Phase Margin (PM) from crossover frequencies.

Note: Maximum error between asymptotic straight lines and the exact curve is 3 dB at the corner frequency for a first-order term.

Measuring Safety: Gain & Phase Margins

Stability metrics indicate the safety buffer before a closed-loop system becomes unstable.
Target margins: GM > 6 dB, PM > 30°.

Gain Margin (GM)

Accelerator Buffer

The factor open-loop gain can increase before instability.

How much faster you can accelerate before losing control.

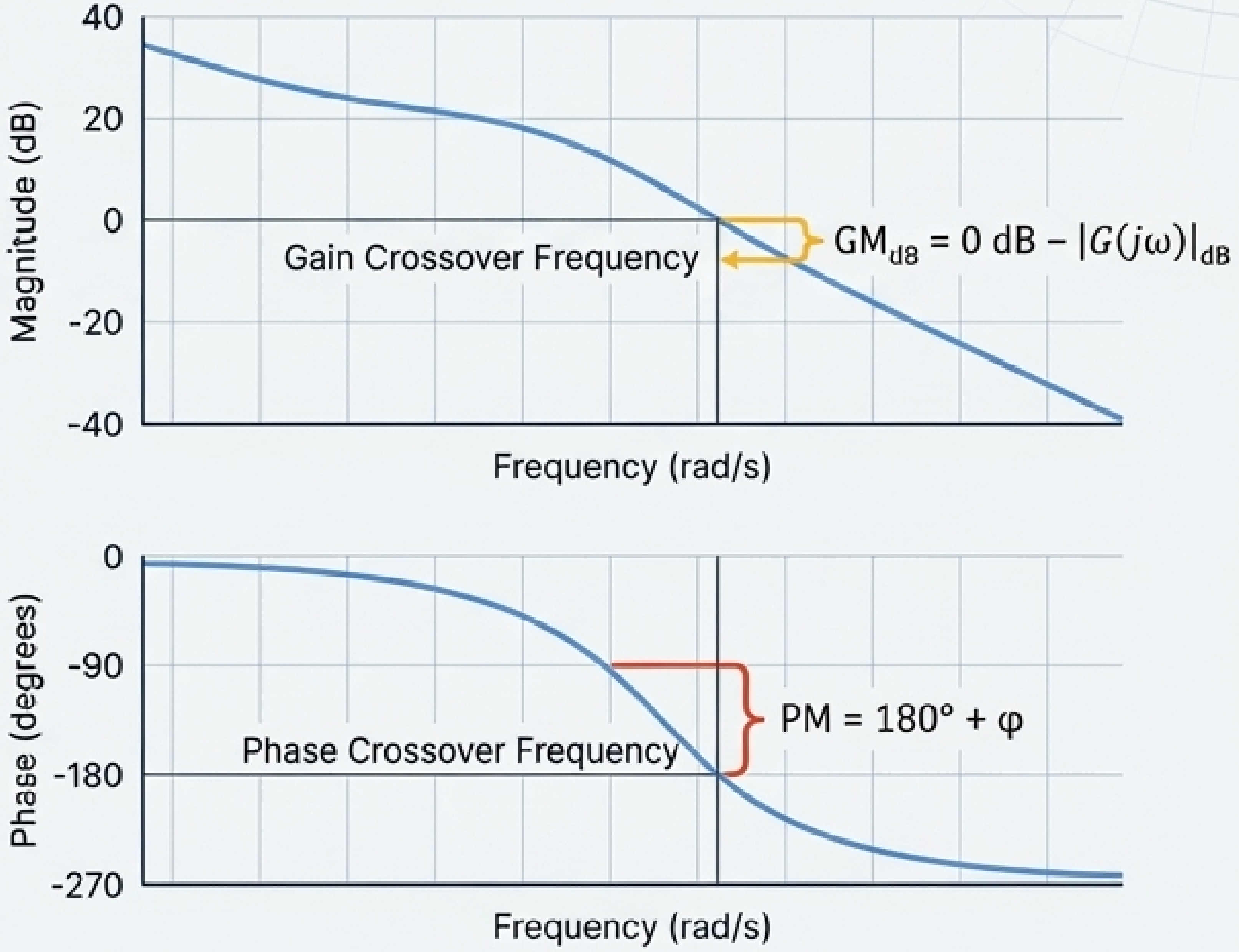
Phase Margin (PM)

Steering Reaction Delay

The additional phase lag tolerated before instability.

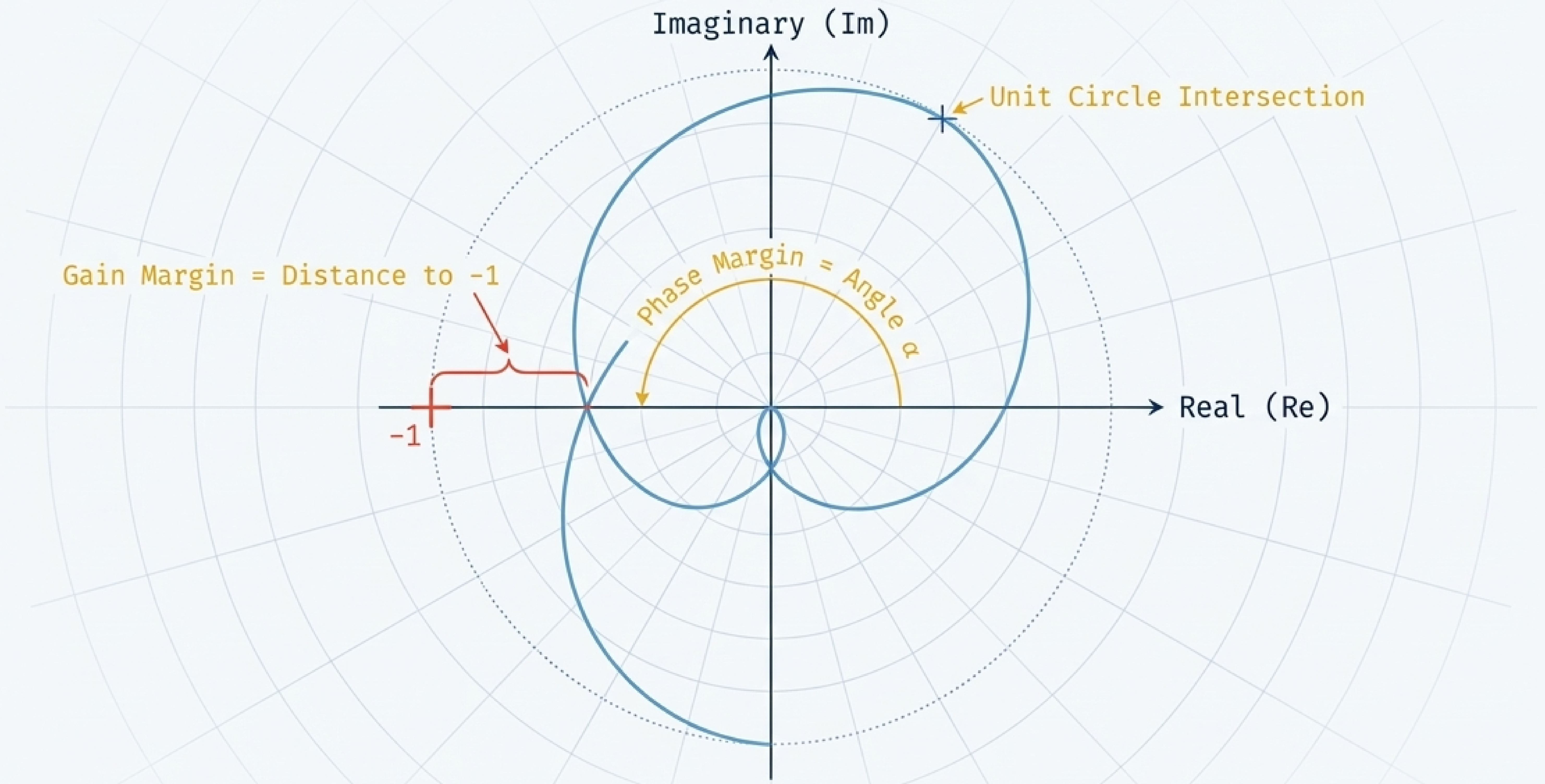
How much delay in reaction time is acceptable before a crash.

“dummy” Bode plot



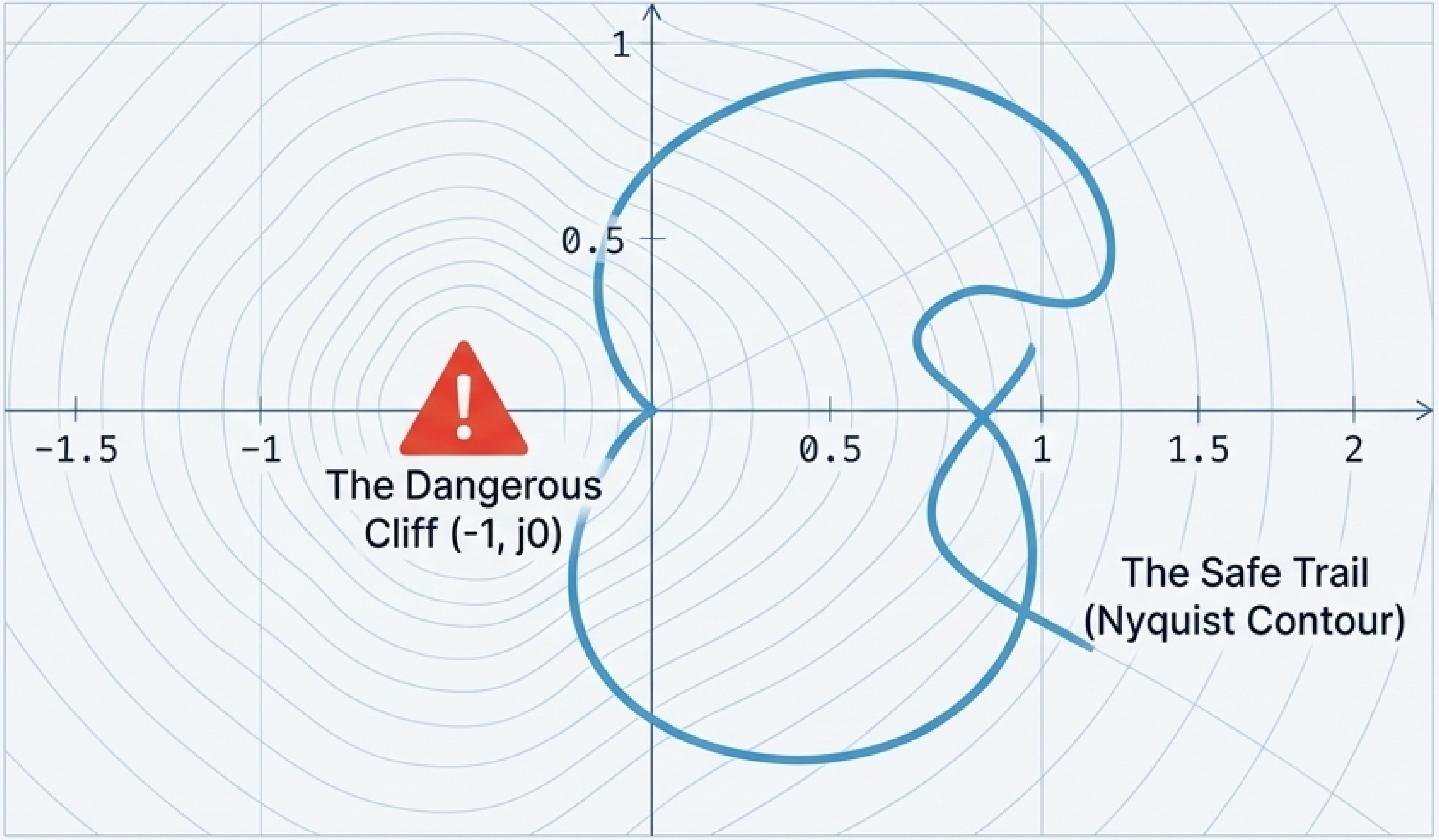
Instrument 2: The Nyquist Diagram

While Bode separates gain and phase, the Nyquist plot combines them into a single parametric curve mapped onto a complex plane, providing a complete, direct assessment of closed-loop stability.



The Hiking Map: Navigating the Contour

The primary goal of Nyquist analysis is to ensure the system's trajectory avoids the critical point of (-1, j0).



Gain Margin

The physical distance between the trail and the edge of the cliff.

Small GM = walking dangerously close to the edge.

Phase Margin

The tolerance for unexpected sharp turns in the trail.

Small PM = sharp turns may throw the system off the cliff.

The Nyquist Stability Criterion

The closed-loop system remains stable if and only if the Nyquist plot of the open-loop transfer function does not encircle the critical point (-1, j0) in a clockwise direction.

$$Z = P + N$$

Z: Number of closed-loop poles in the right-half plane (unstable poles). **Target for stability:** $Z = 0$.

P: Number of open-loop poles in the right-half plane. (Inherent system properties).

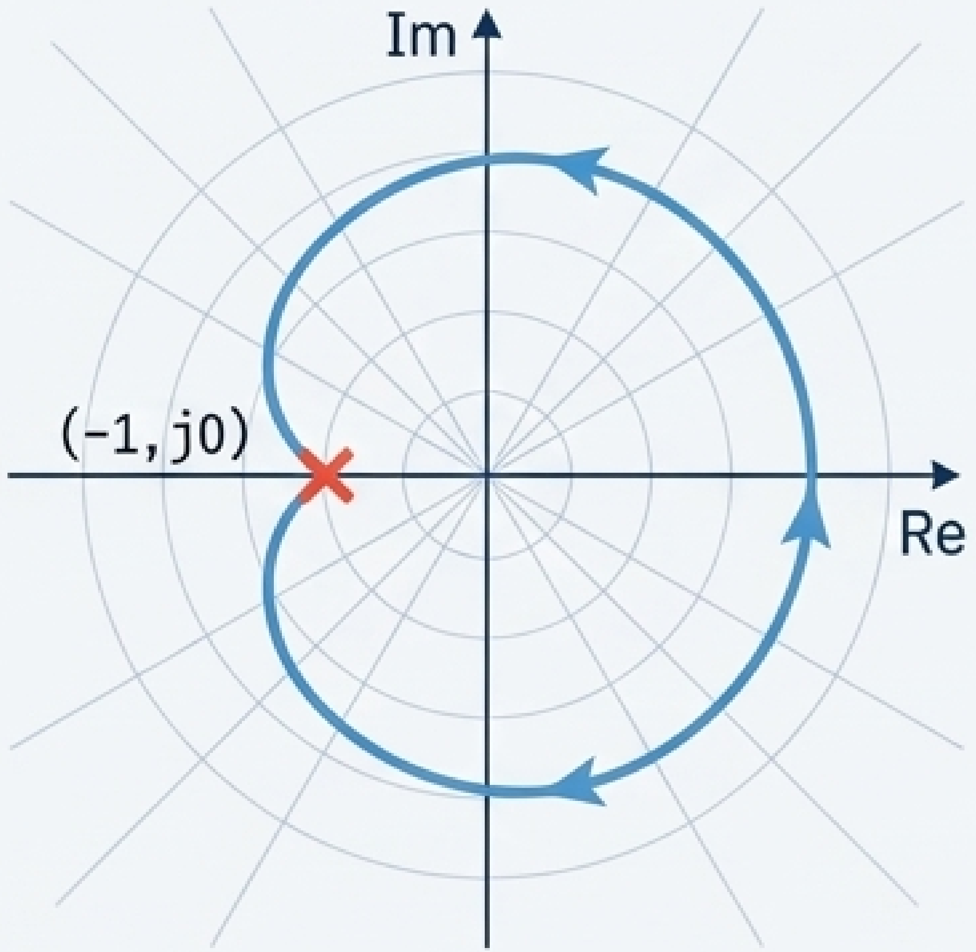
N: Number of clockwise encirclements of the dangerous cliff at (-1, j0). (Graphical observation).

Golden Rule: For an open-loop stable system ($P = 0$), the system is stable ONLY if the Nyquist contour NEVER encircles (-1, j0) ($N = 0$).

Assessing Stability on the Polar Plane

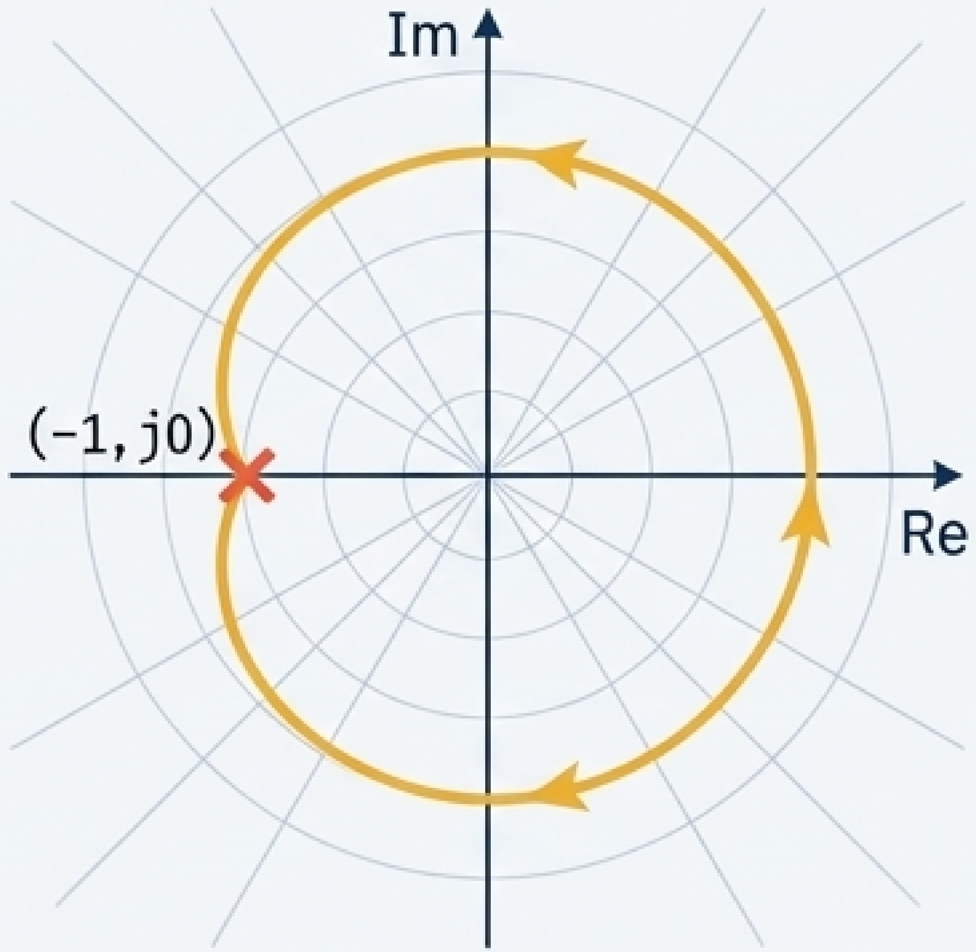
The trajectory of the locus relative to the critical point determines the exact stability state of the system.

State 1: Stable



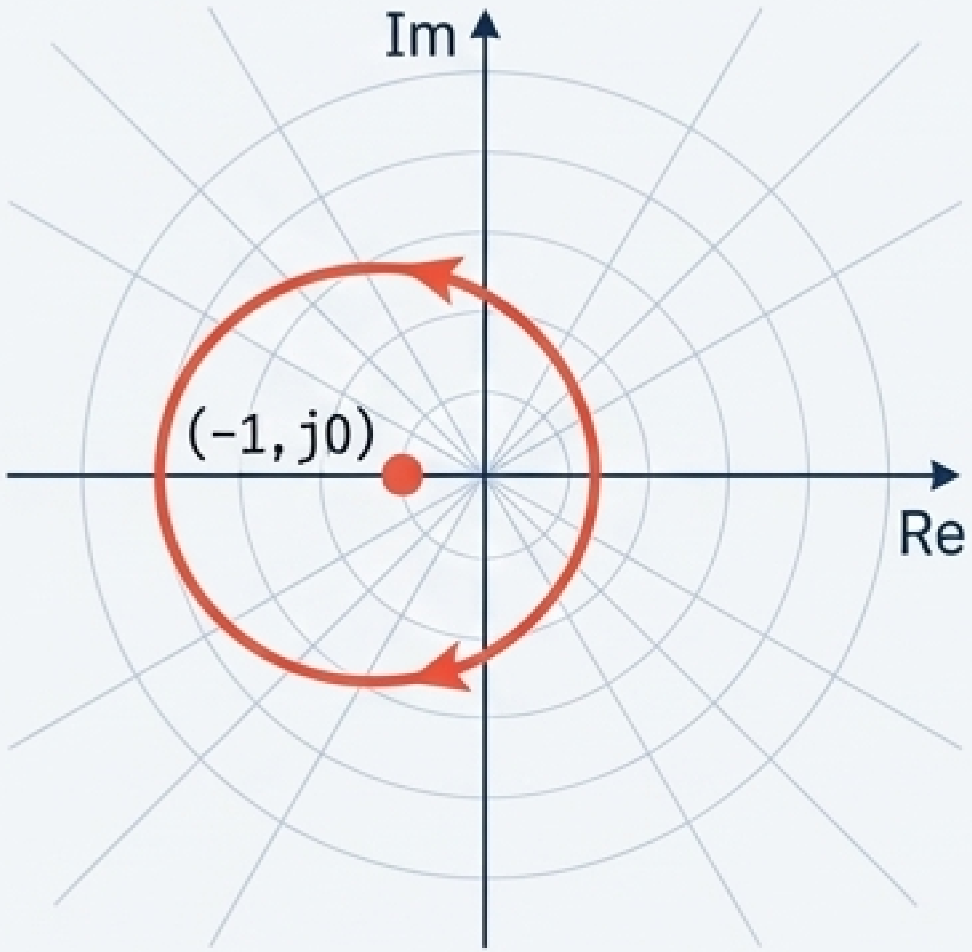
Locus stays to the right of (−1, j0). Oscillation amplitude decreases.

State 2: Marginally Stable



Locus passes exactly through (−1, j0). System oscillates continuously when disturbed.

State 3: Unstable



Locus encircles (−1, j0). Oscillation amplitude aggressively increases.

Synthesis: The Bridge Safety Inspection

Bode and Nyquist plots are not competing methods; they are complementary views of the exact same mathematical truth.

Component-Level Design

Bode plots allow you to check each "component" of the bridge individually. You can easily see the distinct effects of adding a specific pole or a zero to design a controller.

System-Level Verification

The Nyquist diagram tests the overall structural integrity at once. It hides individual components but perfectly reveals the absolute stability of the entire closed-loop system.

The Diagnostic Matrix: Bode vs. Nyquist

A definitive comparison of frequency response methodologies.

Dimension	Bode Plot	Nyquist Diagram
Graph Type	Two separate plots (Mag & Phase)	Single polar plot
Frequency Scale	Logarithmic (ω)	Parametric (0 → ∞)
Core Advantage	Highlights individual poles/zeros	Direct, complete stability test
Core Disadvantage	Stability is less intuitive graphically	Hides individual term contributions
Best Used For	Controller Design & Compensation	Stability Analysis & Verification
MATLAB Cmd	`bode(sys)`	`nyquist(sys)`

The Navigator's Reference: Core Formulas

Standard frequency response mathematics for baseline system modeling.

First-Order System: $G(s) = K / (\tau s + 1)$

Magnitude: $|G(j\omega)| = \dfrac{K}{\sqrt{(\tau\omega)^2 + 1}}$

Phase: $\varphi = -\tan^{-1}(\tau\omega)$

Rule: At corner frequency ($\omega = 1/\tau$): $|G| = K/\sqrt{2}$ (-3 dB), $\varphi = -45°$

Second-Order System: $G(s) = K\omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$

Magnitude: $|G(j\omega)| = \dfrac{K\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}$

Phase: $\varphi = -\tan^{-1}\left(\dfrac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)$

Rule: At resonance ($\omega \approx \omega_n$), a smaller damping ratio (ζ) creates a larger resonant peak.