MEC521 Chapter 5: Estimating the fundamental frequency cheat sheet · handbook §5.1–§5.4

Why it matters

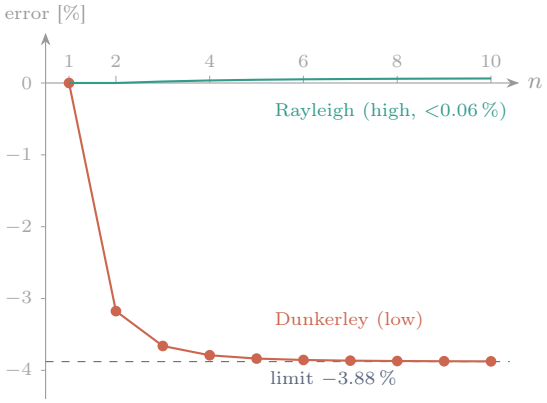
A shaft carrying several discs has as many natural frequencies as masses, and the lowest one sets the critical speed. Two quick methods bracket it without solving the full problem: Dunkerley from below, Rayleigh from above.

Start here

Find the influence coefficients first: a_{ij} is the deflection at i under a unit load at j (F5.1.4, F5.1.5), and $a_{ij} = a_{ji}$ is a free check (F5.1.2). Both methods are built on them.

One picture

Dunkerley always comes out low and Rayleigh high, so together they bracket the exact ω_1 (F5.2.2, F5.3.2). The figure shows how far each misses as the shaft carries more discs; the animation adds the discs one at a time. Animated: `ch5-anim/MEC521-ch5-dun.gif`.

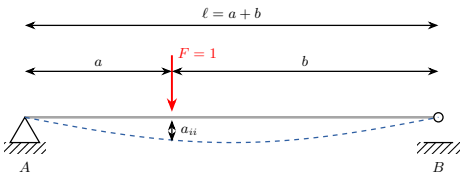


Which formula?

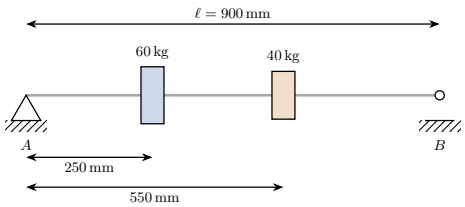
Read the situation first, then pick the formula. The tags are the ones on the examination formula sheet and in Appendix A.

You see...	Use	It holds when
A shaft on bearings carrying discs or pulleys	$a_{ii} = \dfrac{a^2 b^2}{3EI\ell}$ F5.1.4	a and b are the distances from the load to *each bearing*, and $a + b = \ell$. Measuring b from the wrong reference is the commonest error in this material, and the answer still looks plausible.
	$a_{ij} = \dfrac{bx\left(\ell^2 - b^2 - x^2\right)}{6EI\ell}$ F5.1.5	Off-diagonal terms. F5.1.4 is the special case $x = a$.
A quick estimate of the fundamental	$\dfrac{1}{\omega_1^2} \simeq \sum_i m_i a_{ii} = \sum_i \dfrac{1}{\omega_{ii}^2}$ F5.2.1	Fast estimate of the fundamental. Needs only the diagonal terms, so it is the cheapest useful answer in the whole course.
A better estimate, if a shape is available	$\omega_1^2 \simeq \dfrac{g\sum_i m_i w_i}{\sum_i m_i w_i^2}$ F5.3.1	Better than Dunkerley when a sensible trial shape is available. The static shape is usually excellent because it is close to the true mode.
Which side of the true answer am I on?	$\omega_{\text{Dunkerley}} \le \omega_1$ F5.2.2	**Always a lower bound.** So it is conservative for a critical-speed check: if Dunkerley says the shaft is safe with margin, it is.
	$\omega_{\text{Rayleigh}} \ge \omega_1$ F5.3.2	**Always an upper bound**, and exact only if the trial shape is the true mode. Together with F5.2.2 it brackets the answer: $\omega_{\text{Dunk}} \le \omega_1 \le \omega_{\text{Rayl}}$.

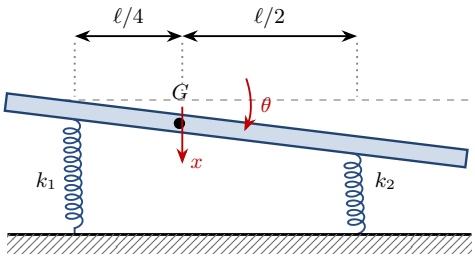
Reading the pictures



The flexibility coefficient of a simply supported shaft (F5.1.4): a and b run from the load to each bearing, and $a + b = \ell$.



The worked example: two discs on one shaft. Dunkerley adds each disc's own frequency as if the other were absent (F5.2.1).



When the coordinates mix a displacement and a rotation, the stiffness matrix is built column by column, one unit displacement at a time.

Ways to lose marks

The wrong bearing. a and b run from the load to each bearing. Measuring both from one end is the commonest error (F5.1.4).
Dunkerley is a lower bound. It discards the coupling, so the shaft looks more flexible and ω_1 comes out low (F5.2.2): the safe side for a critical-speed check.
Rayleigh is an upper bound. Any trial shape but the true mode stiffens the system, so the estimate is high (F5.3.2). The static deflection shape is usually close.
Squares. The formulas give $1/\omega_1^2$ or ω_1^2. Take the root before converting to Hz or rev/min.
An asymmetric $[a]$**.** $a_{ij} = a_{ji}$ always (F5.1.2); an asymmetric flexibility matrix is an arithmetic error.

Where next. Handbook: §5.1–§5.4 for the worked examples, Appendix A for every tag. **Slides:** the Chapter 5 deck, with every animation.