The Dynamic Vibration Absorber

A demonstration · Handbook Example 3.2, animated

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How to use this demonstration

This deck does one job: it shows you *what the absorber does*, before you are asked to compute anything about it.

- The **Chapter 3 deck** derives the absorber and works Example 3.2 by hand. That is where the method lives.
- **This deck** runs that same example — the same machine, the same 20 kg absorber, the same 500 N force — as a simulation you can watch.
- The **handbook** §3.5 is the quick reference you look the result up in afterwards.

Nothing here is a new number

Every value on the next slides is Example 3.2's. They are re-derived through the animation's own code and checked against the handbook by `absorber_demo_verify.m` — 28 checks — so the picture and the worked example cannot drift apart.

The machine, before any algebra

A 200 kg machine sits on a mount of $k = 789.6\,\text{kN m}^{-1}$. It runs at 600 rev/min.

Work out what that running speed is in rad/s:

$$\omega = \frac{2\pi(600)}{60} = 62.83\,\text{rad s}^{-1} \qquad \text{and} \qquad \omega_n = \sqrt{\frac{789\,568}{200}} = 62.83\,\text{rad s}^{-1}$$

So the machine runs *exactly* at its own natural frequency

It is not near resonance. It **is** resonance. Its amplitude does not settle at some large value — it grows, cycle after cycle, for as long as the machine runs.

You cannot change the speed (it is a process requirement) and you cannot change the mount (it is the floor). **What is left?**

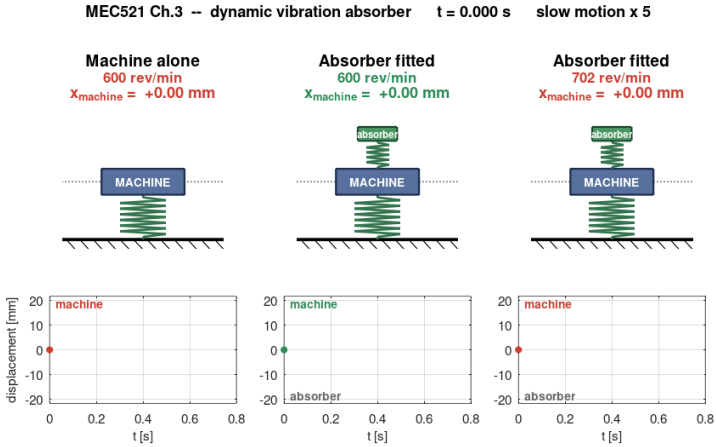
What you are about to watch

Three machines, side by side, driven by the *same* 500 N disturbing force, on the *same* clock.

1. **Machine alone,** 600 rev/min.
   Nothing fitted. Watch the amplitude climb.
2. **Absorber fitted,** 600 rev/min.
   A 20 kg mass on a spring, bolted to the top. Watch the *machine*, not the absorber.
3. **Absorber fitted,** 702 rev/min.
   Identical hardware to panel 2. Only the running speed has changed. This is the part people forget.

The lower trace in each panel is displacement against time — machine and absorber.
Played at one fifth of real speed; the machine really runs at 10 Hz.

The dynamic vibration absorber, running



No response from the controls? Your PDF viewer cannot animate — open `demo-absorber/MEC521-absorber-demo.gif` instead.

Panel 1 — what resonance actually looks like

The machine alone, at 600 rev/min.

There is no steady state. The undamped response at resonance is

$$x(t) = \frac{F_0}{2k}\left(\sin \omega_n t - \omega_n t \cos \omega_n t\right)$$

and the $\omega_n t$ makes the envelope a **straight line through the origin**:

$$|x|_{\max} \approx \frac{F_0\, \omega_n t}{2k}$$

Read it off the animation

After 0.80 s — eight revolutions — the machine is at 15.9 mm and still climbing.

At 8 s it would be at 159 mm. The mount fails long before the algebra does.

Panel 2 — the machine goes still

Bolt on a second spring–mass, tuned so that

$$\omega^2 = \frac{k_a}{m_a} \qquad \text{F3.5.1}$$

With $m_a = 20\,\text{kg}$ at $62.83\,\text{rad}\,\text{s}^{-1}$:

$$k_a = m_a\omega^2 = 79.0\,\text{kN}\,\text{m}^{-1}$$

The absorber then swings

$$X_a = -\frac{F_0}{k_a} = -6.33\,\text{mm} \qquad \text{F3.5.2}$$

What the animation shows

The machine trace is a **flat line at zero**. Not small — **zero**, to five decimal places.

The absorber carries 6.33 mm, in **antiphase** with the force. Its spring pulls back on the machine with exactly F_0, at every instant.

The minus sign in **F3.5.2** *is* the antiphase. It is not a bookkeeping artefact.

Panel 3 — what it cost you

The absorber did not remove the resonance. It **split** it. The combined system has two degrees of freedom, so it has two natural frequencies, and with $\mu = m_a/m_p = 0.10$:

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 + \frac{\mu}{2} \pm \sqrt{\mu + \frac{\mu^2}{4}} = 1.05 \pm 0.3202$$

$$\omega_1 = 53.7\,\mathrm{rad\,s^{-1}}\ (513\,\mathrm{rev/min}), \qquad \omega_2 = 73.5\,\mathrm{rad\,s^{-1}}\ (702\,\mathrm{rev/min})$$

Panel 3 runs the machine at 702 rev/min

Same hardware. Same absorber, still perfectly tuned to 600 rev/min. And the response **grows without bound again** — the absorber mass reaching 19.7 mm, worse than the unprotected machine ever got in panel 1.

The absorber creates a quiet window, and two new cliffs either side of it.

The engineering judgement

Running speed 600 rev/min sits between 513 rev/min and 702 rev/min, with about 15 % margin either way.

- **Acceptable** for a **fixed-speed** machine. It runs in the window and never visits the cliffs.
- **Dangerous** for a **variable-speed** machine — and fatal during **run-up and run-down**, which pass through 513 rev/min every single start.
- Widening the window means a **bigger** μ: the two roots separate as $\sqrt{\mu}$. A heavier absorber buys bandwidth.
- Adding **damping** to the absorber blunts both new peaks — at the price of the machine no longer going exactly still.

That last trade-off is the whole design problem, and it is what Den Hartog's optimum tuning solves.

Three ways to lose marks here

1. **Tuning to the machine's natural frequency instead of the running speed.**
   F3.5.1 is $\omega^2 = k_a/m_a$ where ω is the *excitation*. In this example they happen to be equal — that is why the machine needed an absorber at all — but they need not be, and an exam question will separate them.

2. **Reporting the absorber amplitude without checking there is room for it.**
   6.33 mm of stroke needs 6.33 mm of clearance. State it.

3. **Stopping at "the machine is still".**
   An answer that does not give ω_1 and ω_2 and say where the running speed sits between them has not finished the question. Panel 3 is why.

What the assessment will ask

- Given a machine, a speed and a disturbing force: **size the absorber** and state its stroke. **F3.5.1 F3.5.2**
- Given the fitted absorber: **find the two new natural frequencies** and judge whether the running speed is safely between them.
- Given a *variable*-speed duty: **argue** whether a passive absorber is the right device at all — and say what you would do instead.

Run it yourself

The animation is 200 lines of Octave and needs no add-on packages:

```
cd Handbook/Decks/demo-absorber
octave --no-gui -q --eval "absorber_demo()"
octave --no-gui -q absorber_demo_verify.m
```

Change `P.ma` in `absorber_solve.m` and watch the window widen.

Where this sits

- **Handbook §3.5** — Forced Vibration and the Dynamic Vibration Absorber. The result, stated compactly, with F3.5.1 and F3.5.2 in the formula appendix.
- **Chapter 3 deck** — the derivation, the worked Example 3.2 step by step, and the absorber response curve.
- **Tutorial 3** — the problems. Attempt them after this.
- **This deck** — the picture the other three describe.

Tutorial: MEC521-Tutorial-Ch3, Questions on §3.5.
The answer scheme is released after the session, not with the sheet.