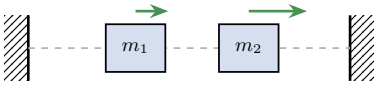
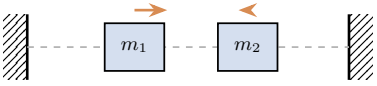
MEC521 Chapter 3: Two-degree-of-freedom systems

Why it matters

Two masses on springs, or one body free to bounce and pitch, have two natural frequencies and two mode shapes. Car suspensions, two-storey frames and vibration absorbers all work this way.



Mode 1: $\omega_1 = 42.8$ rad/s, in phase



Mode 2: $\omega_2 = 70.2$ rad/s, out of phase

Start here

Draw both free-body diagrams, write the equations in matrix form (F3.2.1) and check $[k]$ is symmetric (F3.2.2). Then solve the frequency equation (F3.3.1) and take each mode shape from either row (F3.3.2).

One picture

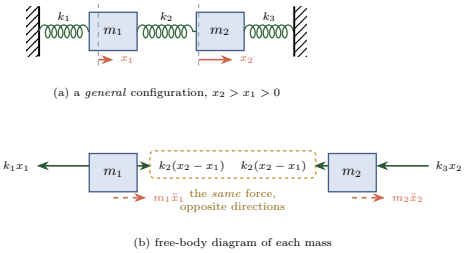
A mode is a pattern, not an amplitude. In each mode both masses move at one frequency in a fixed ratio X_2/X_1: together in the first mode, against each other in the second. Animated: `ch3-anim/MEC521-ch3-modes.gif`.

Which formula?

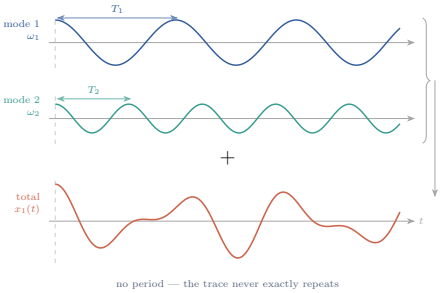
Read the situation first, then pick the formula. The tags are the ones on the examination formula sheet and in Appendix A.

You see...	Use	It holds when
Two masses — write the equations	$[m]\{\ddot{x}\} + [k]\{x\} = \{F\}$ F3.2.1	Any linear multi-degree-of-freedom system.
... and check them before going on	$k_{ij} = k_{ji}, \quad k_{ii} > 0$ F3.2.2	A free check on every assembled matrix. An asymmetric $[k]$ is always an error in the coupling term's sign.
	$\sum_j k_{ij} =$ (stiffness tying mass i to ground) F3.2.3	Row sums: zero for a mass with no path to ground, hence zero for a free end. A non-zero row sum where none is expected means a mis-assembled row.
Find the natural frequencies	$\lvert[k] - \omega^2[m]\rvert = 0$ F3.3.1	The frequency (characteristic) equation. Its roots are ω_i^2.
Find the mode shapes	$\frac{X_2}{X_1} = \frac{k_{11} - \omega_i^2 m_{11}}{-k_{12}} = \frac{-k_{21}}{k_{22} - \omega_i^2 m_{22}}$ F3.3.2	Mode shape. Both rows must give the same ratio — that agreement is your check that ω_i is right.
Nothing anchors it to ground	$\omega_1 = 0$ when $[k]$ is singular F3.3.4	Free–free systems: two rotors on a coupling, a shaft on soft bearings. The zero root is correct physics, not an arithmetic failure.
Kill the vibration at one known speed	$\omega^2 = \frac{k_a}{m_a}$ F3.5.1	**The tuning condition.** Tune the absorber's own natural frequency to the excitation and the primary mass stops dead.
	$X_a = -\frac{F_0}{k_a}$ F3.5.2	Absorber stroke. Sizing the device, and the reason undamped absorbers are large.

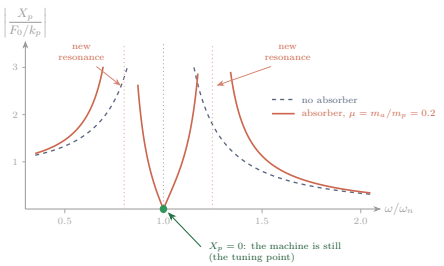
Reading the pictures



Writing the equations (F3.2.1). The coupling spring pulls the two masses with opposite signs, which is why $[k]$ comes out symmetric (F3.2.2).



Released from an arbitrary start, the motion is the sum of both modes, each at its own frequency (F3.4.1). The sum is not periodic, so no single period can be read off the trace.



The absorber (F3.5.1). Tuned to the excitation it stops the machine at one frequency, but leaves two new resonances either side (F3.5.3).

Ways to lose marks

The coupling sign. The coupling force acts on both masses with opposite signs. An asymmetric $[k]$ always means that sign is wrong (F3.2.2).

ω^2, not ω. The roots of the frequency equation are ω_1^2 and ω_2^2. Take the square root before converting to Hz.

Check the mode shape. Both rows must give the same X_2/X_1 (F3.3.2). If they disagree, the frequency is wrong.

An absorber needs a fixed speed. It works at one frequency only; near its two new resonances it makes things worse (F3.5.3).

Where next. Handbook: § 3.1–§ 3.5 for the worked examples, Appendix A for every tag. **Slides:** the Chapter 3 deck, with every animation.