MEC521 Chapter 2: Forced vibration of an SDOF system

Why it matters

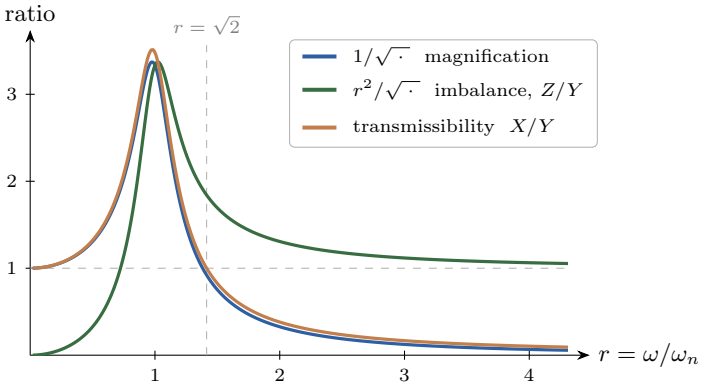
A machine that is pushed keeps moving for as long as the push lasts. Fans, pumps and washing machines shake themselves; cars and instruments are shaken by the ground under them. Every answer in this chapter depends on two numbers only: the frequency ratio $r = \omega/\omega_n$ and the damping ratio ζ.

Start here

Find r first, always. Convert the speed to ω in rad/s ($\omega = 2\pi N/60$ for N rev/min) and work out $\omega_n = \sqrt{k/m}$, then say what r means before any arithmetic: below 1, near 1, or past $\sqrt{2}$.

One picture

One picture holds the chapter. The three ratios share one denominator; the numerator tells you which machine you have. Only the moving-floor curve crosses 1 exactly at $r = \sqrt{2}$. Animated: `ch2-anim/MEC521-ch2-three.gif`.

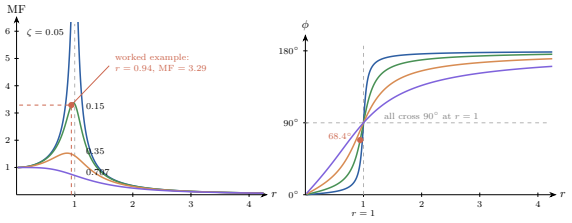


Which formula?

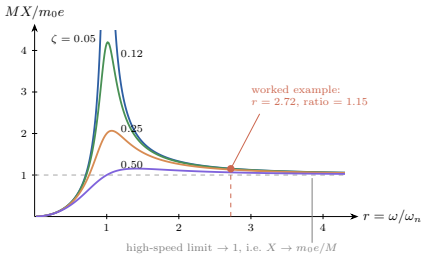
Read the situation first, then pick the formula. The tags are the ones on the examination formula sheet and in Appendix A.

You see...	Use	It holds when
A constant-amplitude force shakes the machine	$X = \dfrac{X_{st}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$, $X_{st} = \dfrac{F_0}{k}$ F2.2.2	Constant-amplitude force. **Not** rotating imbalance — use F2.3.1.
The machine's *own rotation* shakes it	$\dfrac{MX}{m_0 e} = \dfrac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ F2.3.1	Any machine whose excitation comes from its own rotation: fans, pumps, spin dryers. Using F2.2.2 here is the commonest error in Chapter 2.
The floor or base is what moves	$\dfrac{X}{Y} = \sqrt{\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$ F2.4.2	Displacement transmissibility. **Identical in form to the force transmissibility F2.5.1** — one curve answers both questions.
A vehicle crossing a wavy surface	$f = \dfrac{v}{\lambda}$ F2.4.4	Converting road or track geometry into an excitation frequency. Resonance occurs at $v = f_n\lambda$.
How much force reaches the foundation?	$\mathrm{TR} = \dfrac{F_T}{F_0} = \sqrt{\dfrac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$ F2.5.1	Force transmissibility. Same function as F2.4.2.
	$F_T = kX\sqrt{1+(2\zeta r)^2}$ F2.5.2	A second, independent route to F_T once X is known — use it to cross-check F2.5.1.
Will this mount isolate at all?	$\mathrm{TR} = 1$ at $r = \sqrt{2}$ for every ζ F2.5.3	**Isolation is only possible for $r > \sqrt{2}$.** Below it the mount amplifies, whatever damper you fit.

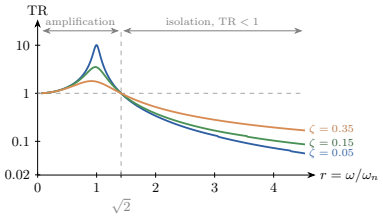
Reading the curves



A constant force (F2.2.2, F2.2.3). Damping matters only near $r = 1$, and every phase curve crosses 90° at $r = 1$, whatever the damping. The marked point is the worked example in §2.2.



A spinning mass (F2.3.1). It starts at zero and levels off at one: far above resonance the amplitude settles at the imbalance's own floor (F2.3.2).



Isolation (F2.5.1, F2.5.3). Every curve crosses 1 at $r = \sqrt{2}$. To the left damping helps; to the right the order reverses.

Ways to lose marks

The wrong numerator. Three formulas share one denominator. Name the ratio before you use it: a constant force is F2.2.2, a spinning mass F2.3.1, a moving floor F2.4.2.

The phase above resonance. For $r > 1$ the calculator's $\tan^{-1}$ gives a negative angle. The lag is in the second quadrant: add 180° (F2.2.3).

$\sqrt{2}$ is a minimum. Isolation needs $r > \sqrt{2}$, so the mount stiffness is a maximum. A softer mount isolates; a stiffer one fails the specification (F2.5.3).

Damping past $\sqrt{2}$. Above $r = \sqrt{2}$ more damping lets more through. Damping is for getting through resonance on the run-up, not for isolation (F2.5.1).

rev/min is not rad/s. N rev/min gives $\omega = 2\pi N/60$. Using N or f as ω puts r out by a factor of 2π or more.

Where next. Handbook: §2.1–§2.5 for the worked examples, Appendix A for every tag. **Slides:** the Chapter 2 deck, with every animation.