Why Vibration?

The Tacoma Narrows Bridge, 7 November 1940

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MEC521 Vibrations · Week 1 opening lecture

Semester 20264

Before we start — vote

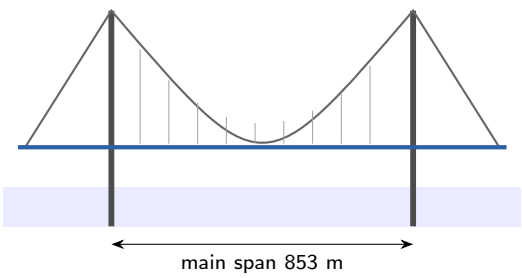
A bridge is strong enough to carry every lorry you can put on it.

Can a steady wind of **68 km/h** — a blustery afternoon — bring it down?

A	**B**	**C**
No. It is strong.	Yes, if it resonates.	Something else.

Hold on to your answer. By the end of this hour most of you will change it — including those who picked B.

The bridge



- Puget Sound, Washington, USA. Opened **1 July 1940**.
- Third-longest suspension span in the world.
- Two lanes; deck only **11.9 m** wide.
- Stiffened by solid plate girders only **2.4 m** deep.
- Cost $6.4 million.

Slenderness

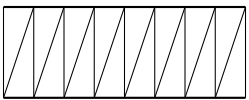
width : span = 1 : 72

depth : span = 1 : 350

Nothing that long had ever been built that thin.

A design decision

Eldridge's design



open truss — wind passes through

Moisseiff's design (built)



solid H-section — wind must go round

- The shallow girder was lighter, cheaper and more elegant. It relied on the cables for stiffness (*deflection theory*).
- It passed **every static check**: dead load, traffic load, and a static wind pressure.
- Nobody asked a *dynamic* question: what does wind do to a structure that can move?

“Galloping Gertie”

From the day it opened, the deck rose and fell in long waves in quite moderate wind.

- Motorists drove over it for the thrill of it. Cars ahead would disappear into a trough.
- Engineers added tie-down cables and hydraulic buffers. The tie-downs snapped; the buffers were disabled when sandblasting damaged their seals.
- Professor F. B. Farquharson, University of Washington, was already testing a scale model in a wind tunnel to find a fix.

Four months

It opened on 1 July. It fell on 7 November.

Thursday 7 November 1940

early morning	Wind about **19 m/s** (42 mph), steady. The deck is moving vertically, as it often did.
≈ 10:00	A cable band at mid-span slips. The motion **changes character**: the deck starts to **twist**, one half up while the other half goes down, the centre line still. The bridge is closed to traffic.
	Leonard Coatsworth abandons his car on the deck and crawls to safety. His daughter's dog, Tubby, is still inside. Farquharson tries to reach it and cannot.
≈ 11:00	The twisting grows until the hangers fail. About 180 m of deck, then the whole main span, falls into the Narrows.

No human was killed. That was luck, not engineering.

Watch it — and count

The collapse was filmed by Barney Elliott and Harbine Monroe. The film is in the US National Film Registry and in the public domain.

Play the footage

`archive.org/details/CEP176` (2 min, silent)

With Farquharson's wind-tunnel model: `archive.org/details/uwlibraries_oclc892993126_tacoma`

Your job while it plays — three things, write them down:

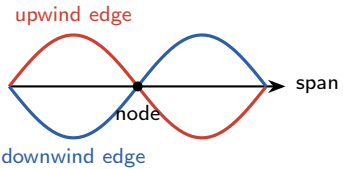
1. **How often?** Count the twists in one minute.
2. **Where is it still?** Find the line on the deck that does not move.
3. **Is it steady?** Is the amplitude constant, or growing?

What you should have counted

1. About **12 twists per minute**:

$$f = \frac{12}{60\ \text{s}} = 0.2\ \text{Hz}, \qquad T = 5\ \text{s}$$

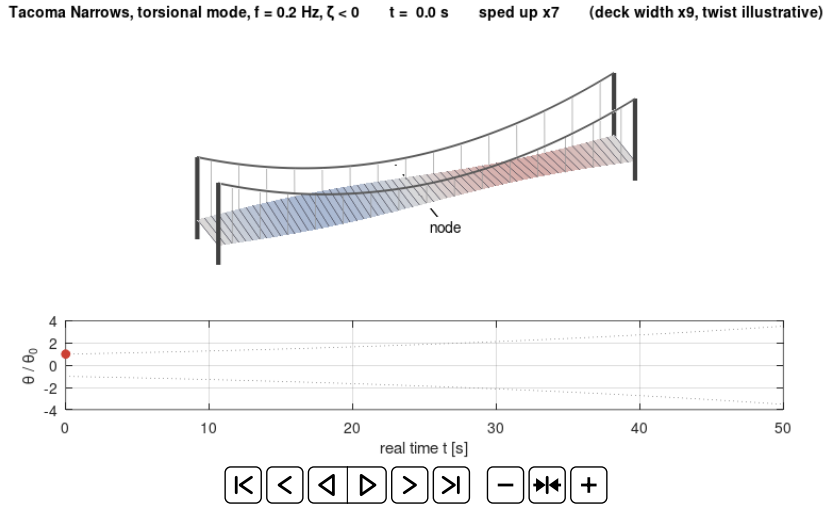
2. The **centre line** at mid-span barely moves — a *node*. Each half twists the opposite way.
3. The motion **grew** until something broke.



The two edges of the deck, seen from the side.

You have just measured a **natural frequency** and a **mode shape**. That is Chapter 1 and Chapter 3 of this course.

The torsional mode, reconstructed



One torsional mode at 0.2 Hz, amplitude growing. Width and twist exaggerated. No response from the controls? Open `tacoma-anim/MEC521-tacoma-bridge.gif`.

The textbook story is wrong

Most physics textbooks say: *the wind shook the bridge at its natural frequency — resonance.* Test it.

Wind past a bluff body sheds vortices at the **Strouhal frequency** (Fluid Mechanics):

$$f_{vs} = \frac{St\,U}{D} = \frac{0.11 \times 19}{2.4} \approx 0.87 \text{ Hz}$$

$St \approx 0.11$ for the H-section; D = girder depth (Billah & Scanlan, 1991).

Compare

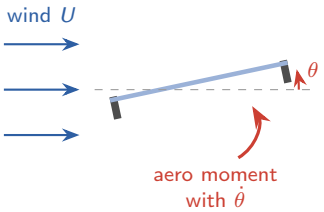
shedding	≈ 0.87 Hz
bridge	0.2 Hz

ratio $\approx$ **4.4**

Not resonance

The forcing was more than four times too fast, and a steady wind is not a periodic force anyway. Something else fed the motion.

Self-excitation: the motion makes its own force



1. The deck twists by θ.
2. The wind now meets it at an angle; the flow separates and re-attaches differently on top and bottom.
3. The resulting moment *lags* the twist, so it pushes **in the direction the deck is already moving** — proportional to $\dot{\theta}$.
4. Each cycle the wind does **positive work** on the deck.

The name for this

Aeroelastic flutter (here, single-degree torsional flutter). The force exists only because the structure moves.

Write it as the equation you will learn in Week 2

Newton's second law for rotation of the deck section:

$$I\ddot{\theta} \;+\; c_s\,\dot{\theta} \;+\; k_\theta\,\theta \;=\; \underbrace{c_a(U)\,\dot{\theta}}_{\text{aero moment}}$$

Move the wind term to the left:

$$I\ddot{\theta} \;+\; \big(c_s - c_a(U)\big)\dot{\theta} \;+\; k_\theta\,\theta \;=\; 0$$

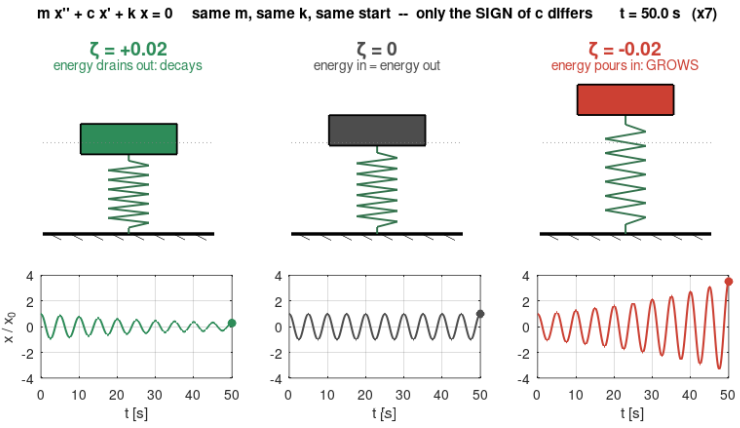
- Low wind: $c_s > c_a$. Net damping positive. Motion dies out.
- Above the **flutter speed**: $c_a > c_s$. Net damping **negative**.

Negative damping

$$\theta(t) = \Theta\, e^{-\zeta\omega_n t}\cos\omega_d t, \quad \zeta < 0$$

The exponential *grows*. There is no steady state — only failure.

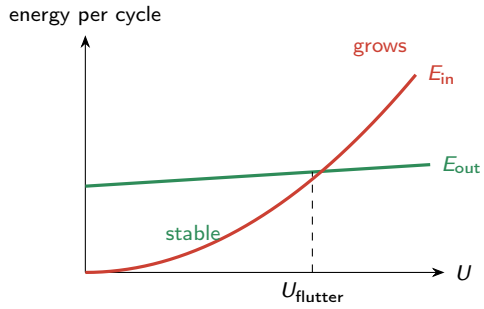
The whole story in one sign



After 10 cycles: $\zeta = +0.02$ leaves **0.28** of the start, $\zeta = -0.02$ leaves **3.5 times** the start.

$\zeta = \pm 0.02$ is illustrative, not measured. GIF: `tacoma-anim/MEC521-tacoma-zeta.gif`.

Where the energy comes from



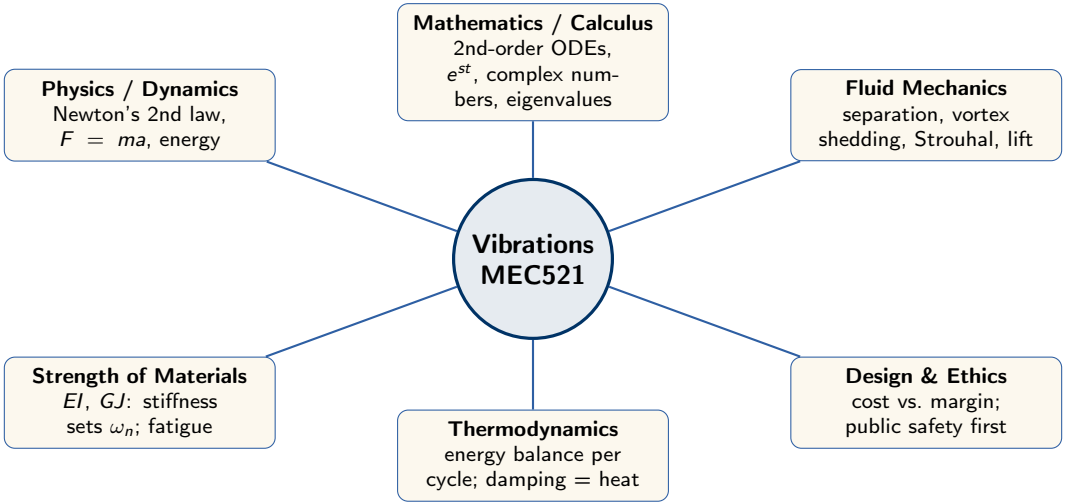
Illustrative curves, not Tacoma data.

Look at it as an **energy balance** (Thermodynamics' first law, applied to one cycle):

$$\Delta E_{\text{cycle}} = E_{\text{in, wind}} - E_{\text{out, damping}}$$

- Damping turns motion into heat. That is *all* damping is.
- Below U_{flutter}: the balance is negative. Safe.
- Above it: every cycle stores more energy than the last, until a member breaks.

Every subject you have taken was on that bridge



Vibration is where they stop being separate subjects.

Strength of Materials: why the H-section was so weak in twist

Open thin-walled section



Tacoma 1940: two plate girders + deck

$$J \approx \tfrac{1}{3} \sum b\, t^3$$

t^3 with t in *millimetres* — tiny.

Closed box section



$$J = \frac{4A_m^2}{\oint ds/t}$$

grows with the *enclosed area* A_m^2 — orders of magnitude stiffer.

Why it matters here

Low $GJ \Rightarrow$ low torsional stiffness $k_\theta \Rightarrow$ low torsional ω_n *and* a low flutter speed. The same girder that was "strong enough" was barely stiff enough to resist twist at all.

What engineers did next

- **1941:** Federal Works Agency board — Ammann, von Kármán, Woodruff — reports that aerodynamic forces had never been considered in suspension-bridge design.
- **1950:** The replacement opens with a deep, **open stiffening truss** and wind-tunnel-tested aerodynamics.
- **Today:** every long-span bridge is tested as an *aeroelastic* model in a wind tunnel before it is built. Many use a streamlined **closed box** deck: stiff in torsion, and the wind flows round it.

The lesson that stuck

Static strength is *necessary*. It is not *sufficient*. A structure that can move must be designed for how it moves.

Tacoma killed no one. These did.

Event	Vibration mechanism	Deaths	In MEC521
Lockheed Electra, 1959–60	whirl-mode flutter of an engine nacelle tore off a wing, twice	97	Ch.1, Ch.3
Mexico City earthquake, 1985	soft lake-bed soil amplified shaking at ≈ 2 s period; buildings near that period collapsed	$>5\,000$	Ch.2 base motion
Eschede ICE train, 1998	fatigue crack in a rubber-sprung wheel tyre; the tyre broke at 200 km/h	101	Ch.2, Ch.4
Sayano–Shushenskaya, 2009	turbine unit with a record of high vibration; cover bolts failed in fatigue, turbine hall flooded	75	Ch.4 monitoring

In every one of these the parts were *strong enough*. What failed was the understanding of how they *moved*.

And one that was caught in time

London Millennium Footbridge, June 2000

- Swayed sideways as soon as crowds walked on it. Pedestrians fell into step with the sway and made it worse — another self-excited system.
- Closed **two days** after opening.
- Fixed by adding damping, not strength: **37** viscous dampers and **52** tuned-mass dampers (89 in all). Reopened 2002.

That fix is in this course

The tuned-mass damper is the *dynamic vibration absorber* of Chapter 3. You will design one.

What MEC521 gives you, chapter by chapter

Ch.1 SDOF free vibration	ω_n, damping ratio ζ, and what happens when ζ changes sign. *Tacoma lives here.*
Ch.2 SDOF forced vibration	resonance, rotating imbalance, base motion, isolation. *Mexico City, every pump on a baseplate.*
Ch.3 Two-DOF systems	mode shapes, nodes, the vibration absorber. *The Tacoma twist; the Millennium Bridge fix.*
Ch.4 FRF & analysis	measure it, read the spectrum, catch the fault early. *Sayano–Shushenskaya.*
Estimation methods	Rayleigh and Dunkerley: estimate ω_n *before* you build.

By Week 14 you will be able to explain the Tacoma collapse with your own equations, not someone else's story.

Three ideas to unlearn today

1. **"Strong means safe."**
   Tacoma was strong. Strength is a static answer to a dynamic question.
2. **"Tacoma was resonance."**
   It was self-excited flutter. Resonance needs a periodic force near ω_n; flutter needs only a steady wind and negative net damping.
3. **"Damping is a small correction."**
   At Tacoma the *sign* of the damping term decided everything. At the Millennium Bridge, damping *was* the fix.

Now vote again — and discuss with your neighbour (5 min)

1. Look back at your vote. **A**, **B** or **C** — and why?
2. The 1950 bridge used a much deeper, stiffer deck. Using $I\ddot{\theta} + c\,\dot{\theta} + k_\theta\theta = 0$, name *two* ways that change helped.
3. Name one machine or structure you might work on after graduating — a pump, a fan, a turbine, a drone, a car — and say what could make it vibrate.
4. Moisseiff's design saved money. **Who** should decide how much margin is enough, and on what evidence?

The engineer's first duty

Every professional code, the Board of Engineers Malaysia's included, puts the safety of the public first. Knowing how things vibrate is part of that duty.

Next: Chapter 1

$$m\ddot{x} + c\dot{x} + kx = 0$$

The equation that would have predicted Tacoma.
We start by deriving it from a free-body diagram.

- Slides: `MEC521-Ch1-SDOF-Free-Vibration.pdf`
- Handbook: Chapter 1, §1.1–1.2 before next class.

Sources

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- K. Y. Billah, R. H. Scanlan (1991). Resonance, Tacoma Narrows bridge failure, and undergraduate physics textbooks. *American Journal of Physics* 59(2), 118–124.
- Washington State Department of Transportation. *Tacoma Narrows Bridge history*.
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- P. Dallard et al. (2001). The London Millennium Footbridge. *The Structural Engineer* 79(22), 17–33.
- US Civil Aeronautics Board reports, Braniff Flight 542 (1959) and Northwest Flight 710 (1960).
- S. S. Rao. *Mechanical Vibrations*, course textbook.

Derived numbers: `tacoma_verify.py` (13 checks); animations: `tacoma-anim/tacoma_anim_verify.m` (14 checks).