MEC521 Chapter 1: Free vibration of an SDOF system

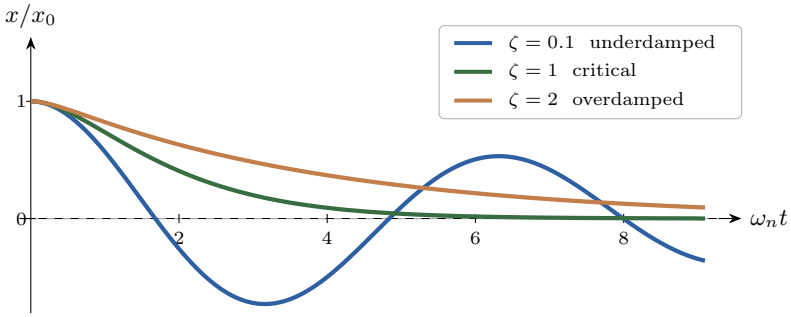
Why it matters

Pull a mass on a spring and let go: it swings at its own natural frequency while damping drains the energy away. That frequency is the number every later chapter compares against.

Start here

Model first: one mass m, one equivalent stiffness k, one damper c, with displacement measured from static equilibrium so the weight drops out. Then $\omega_n = \sqrt{k/m}$ (F1.3.2) and $\zeta = c/c_c$ with $c_c = 2\sqrt{km}$ (F1.4.2, F1.4.3).

One picture



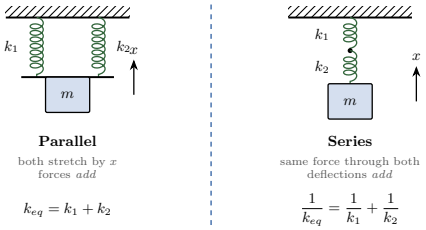
The same system released from the same place at three damping ratios. Below $\zeta = 1$ it oscillates inside a shrinking envelope; at $\zeta = 1$ it returns fastest without overshoot; above 1 it creeps back more slowly, not more safely. Animated: `ch1-anim/MEC521-ch1-regimes.gif`.

Which formula?

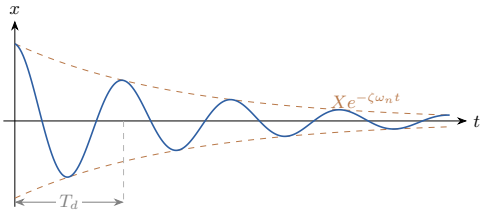
Read the situation first, then pick the formula. The tags are the ones on the examination formula sheet and in Appendix A.

You see...	Use	It holds when
Two or more springs carrying one mass	$k_{\text{eq}} = \sum_i k_i$ F1.2.1	Springs in **parallel** — they share the deflection, not the load. Physical layout is irrelevant.
	$\frac{1}{k_{\text{eq}}} = \sum_i \frac{1}{k_i}$ F1.2.2	Springs in **series**. The softest element dominates.
The mass sits on a beam, arm or shaft	$k = \frac{3EI}{L^3}$ F1.2.3	Mass at the free end of a cantilever; beam mass negligible.
How fast does it oscillate?	$\omega_n = \sqrt{\frac{k}{m}}$ F1.3.2	The single most-used result in the course.
A measured decay trace — how much damping?	$\delta = \frac{1}{n} \ln \frac{x_i}{x_{i+n}}$ F1.5.1	Viscous damping only — see F1.5.4. Use $n \geq 3$ where the trace allows.
	$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$ F1.5.2	Always valid for viscous damping. Use this form, not the approximation, unless asked.
How long before it stops moving?	$t_{5\%} = \frac{\ln 20}{\zeta \omega_n}$ F1.4.7	Settling time. Note it depends on the *product* $\zeta\omega_n$, so stiffening and damping are both routes to a faster settle.
The decay is straight, not curved	$\Delta x_{\text{per cycle}} = \frac{4F_d}{k}$ F1.5.4	**Coulomb** (dry friction) damping. Its decay is linear, not exponential: constant *differences*, not constant ratios. If the ratios are not constant, F1.5.1 does not apply.

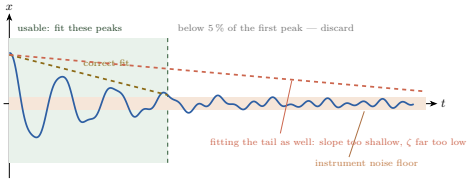
Reading the pictures



Springs that share the displacement are in parallel and add (F1.2.1); springs that share the force are in series and the softest dominates (F1.2.2). Decide by the question, not the drawing.



The underdamped response oscillates at ω_d, a little slower than ω_n (F1.4.4), inside an envelope that decays at $\zeta\omega_n$.



Damping from a record (F1.5.1). Peaks lost in the noise stop decaying, and fitting through them makes ζ come out too small: use clean peaks, $n \geq 3$ apart where the trace allows.

Ways to lose marks

rad/s is not Hz. ω_n is in rad/s. The frequency in Hz is $f_n = \omega_n/2\pi$ and the period is $T = 1/f_n$. Mixing them is the commonest unit error.

Weight left in the equation. Measured from static equilibrium, the weight cancels the spring's pre-stretch. If mg is still in your equation of motion, the coordinate is wrong.

More damping is not safer. Above $\zeta = 1$ the system creeps back more slowly. Critical damping is the fastest return without overshoot.

A straight decay. If the peaks fall by equal differences, not equal ratios, the damping is dry friction (F1.5.4) and the logarithmic decrement does not apply.

Where next. Handbook: §1.1–§1.5 for the worked examples, Appendix A for every tag. **Slides:** the Chapter 1 deck, with every animation.